Principles of Communications
EES 351

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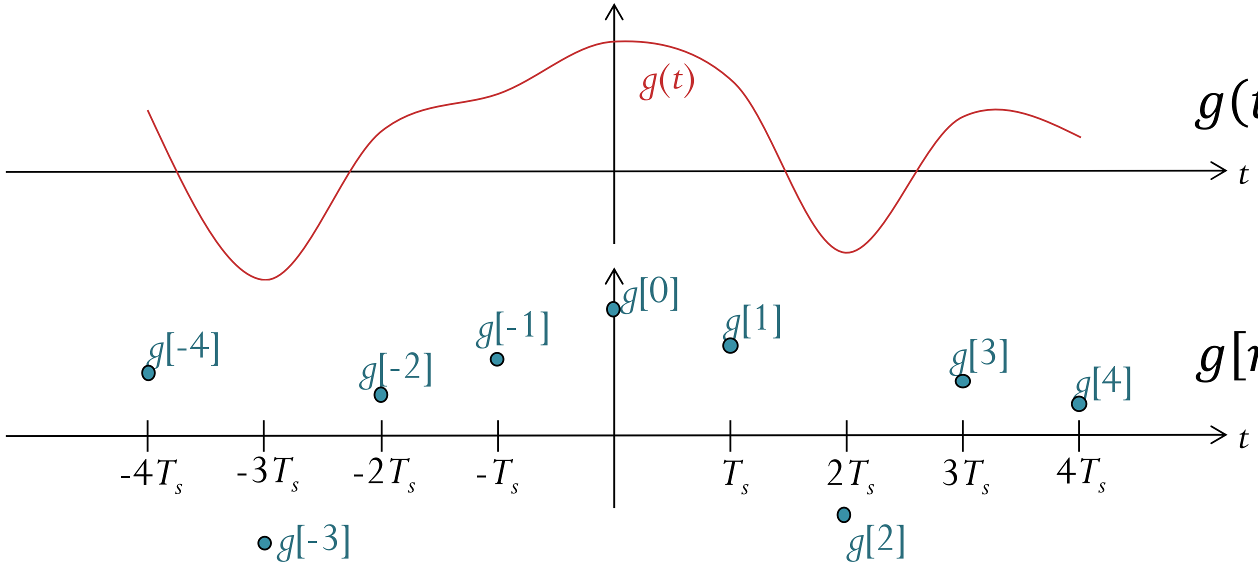
6.3 Reconstruction

Sampling and Reconstruction

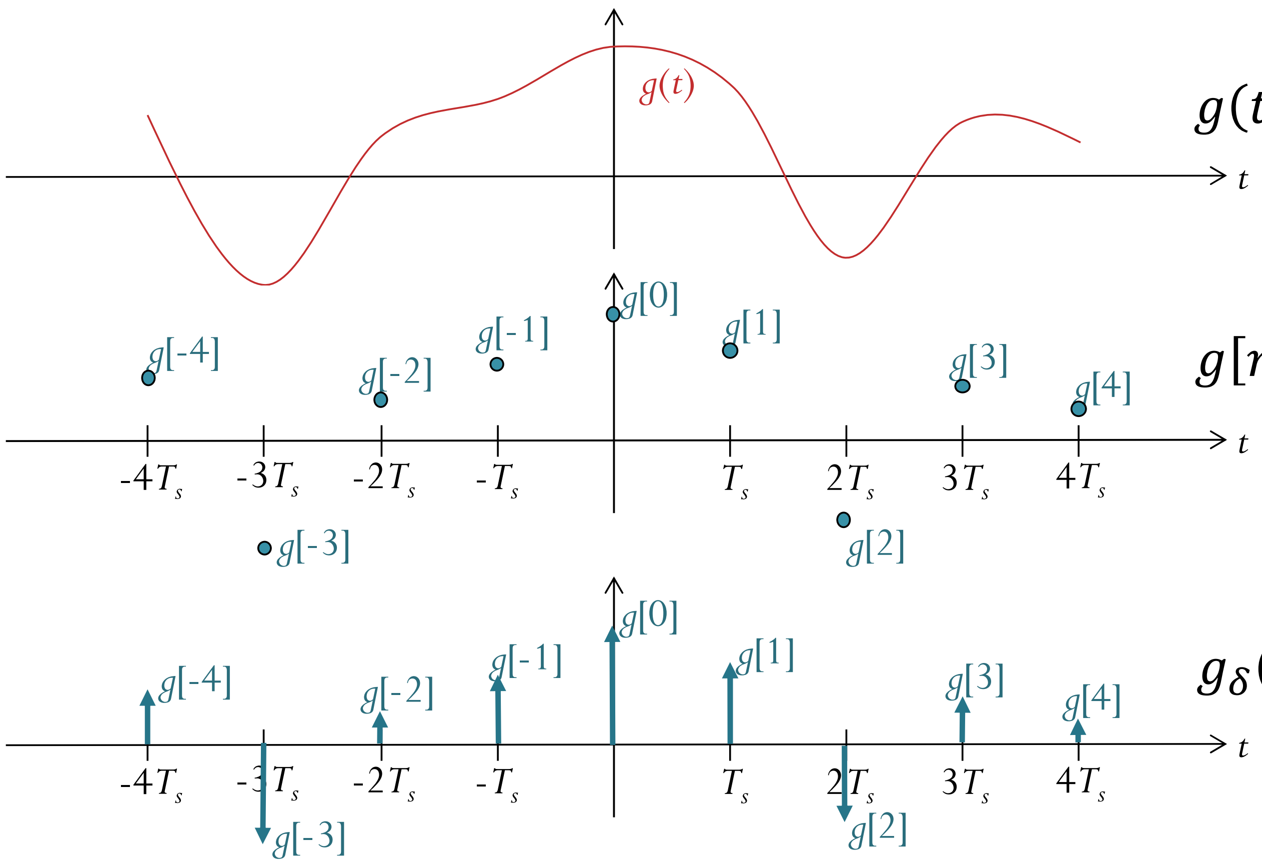


$g(t)$

$g[n] = g(nT_s)$

How can we get $g(t)$ back from $g[n]$?

Review: Sampling



$g(t)$

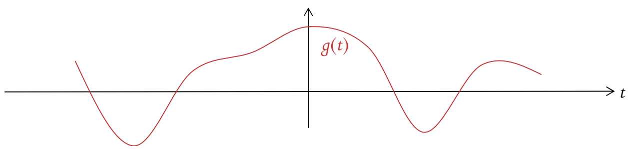
$g[n] = g(nT_s)$

$g_\delta(t) = \sum_{n=-\infty}^{\infty} g[n]\,\delta(t - nT_s)$

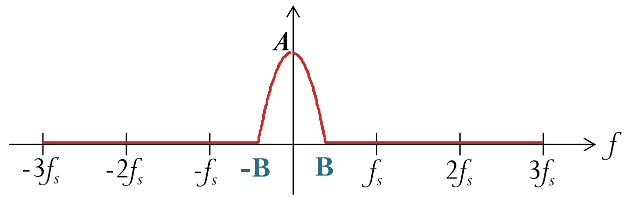
To get a better understanding of the sampling operation, we define $g_\delta(t)$.

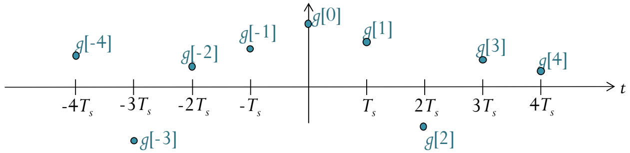
Sampling

Time Domain | Frequency Domain

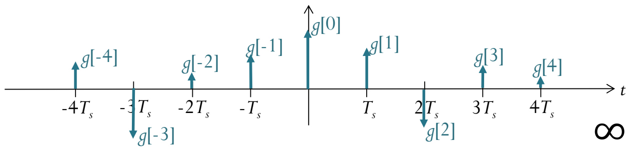


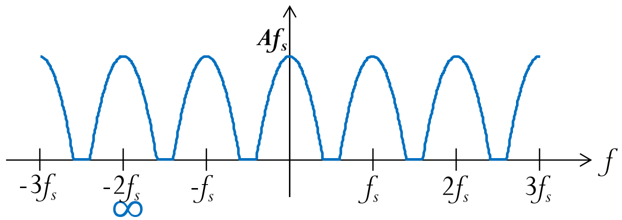
$$g(t) \overset{\mathcal{F}}{\rightleftharpoons} G(f)$$





$$g[n] = g(nT_s)$$

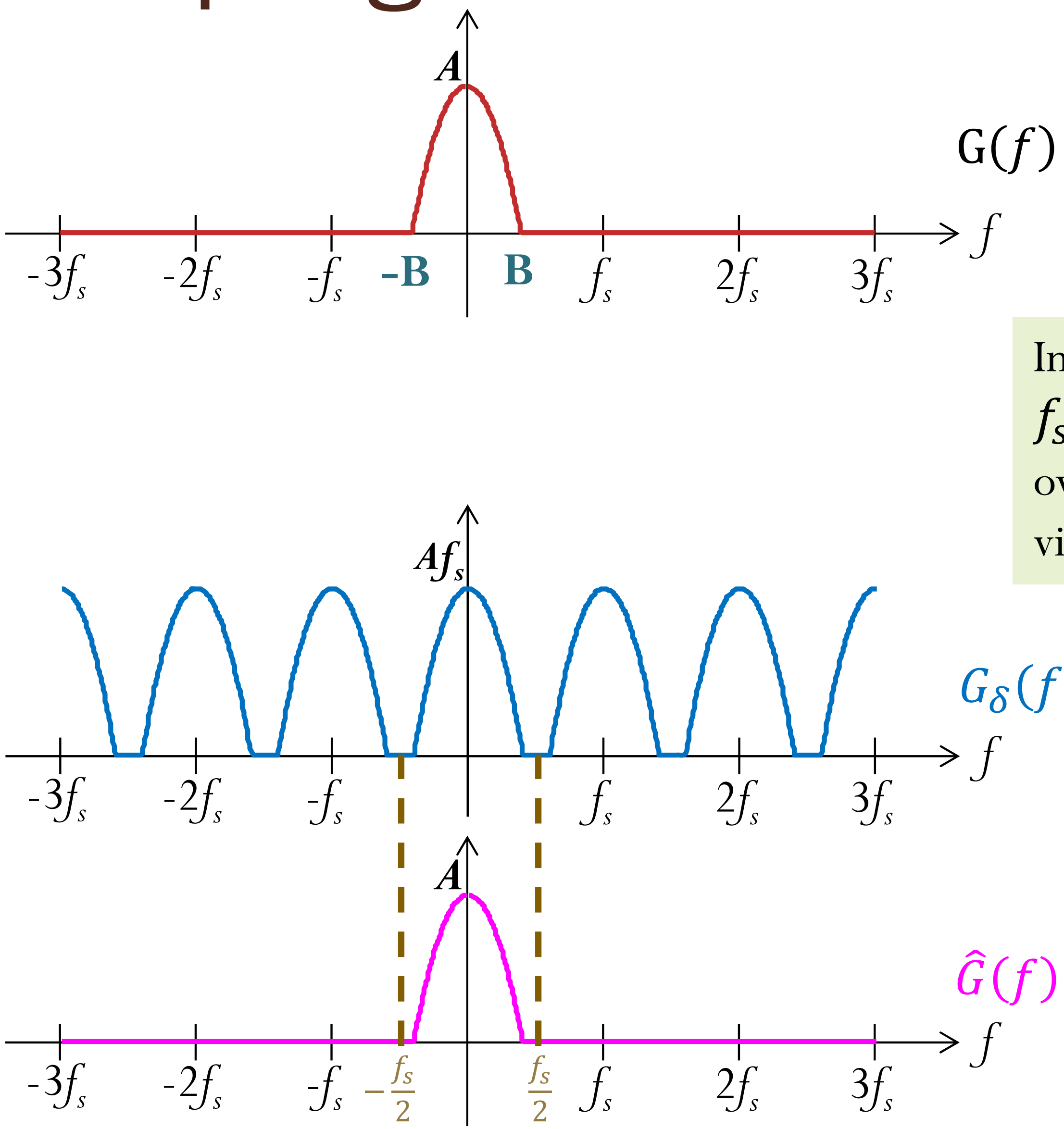




$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g[n]\,\delta(t - nT_s) \overset{\mathcal{F}}{\rightleftharpoons} G_\delta(f) = \sum_{k=-\infty}^{\infty} f_s G(f - kf_s)$$

In Section 6.2, we saw the frequency-domain effect of the sampling operation

Sampling and Reconstruction



In Section 6.2, we saw that, when $f_s > 2B$, the replicas do not overlap. The original $G(f)$ is clearly visible inside $G_\delta(f)$.

$$G_\delta(f) = \sum_{k=-\infty}^{\infty} f_s G(f - k f_s)$$

With **filtering**, we may be able to "extract" $G(f)$ from the ideal sampled signal $G_\delta(f)$.

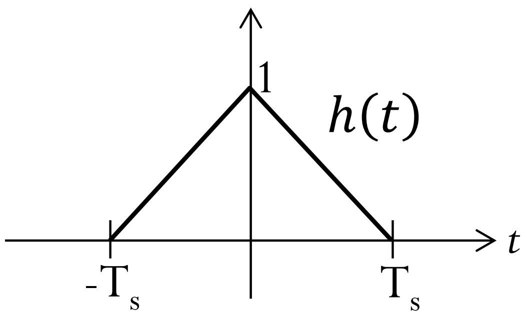
Reconstruction and Convolution

In the time domain, filtering $g_\delta(t)$ is simply the same as convolving it with another signal $h(t)$.

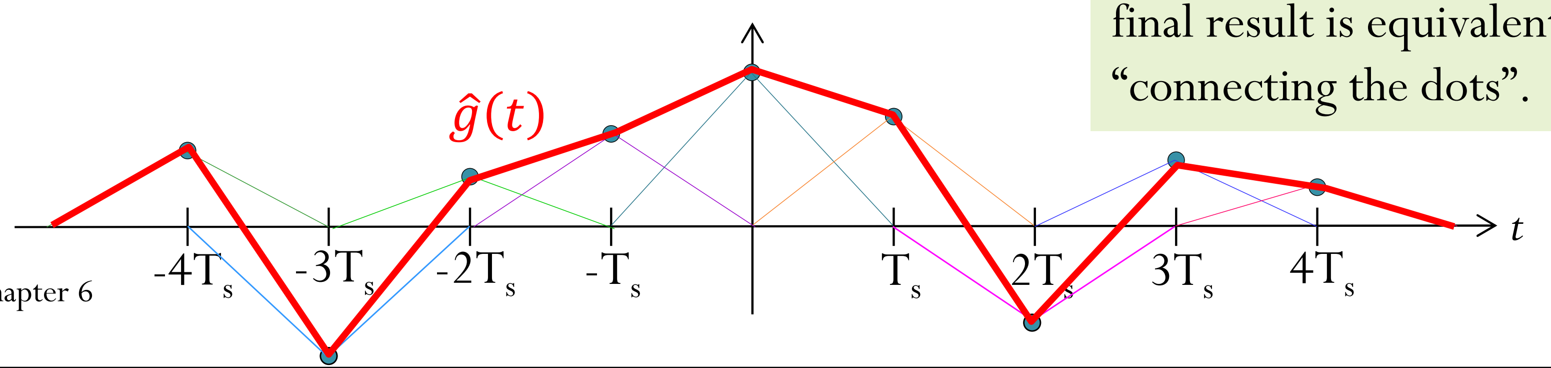
$$\hat{g}(t) = g_\delta(t) * h(t) = \left(\sum_{n=-\infty}^{\infty} g[n]\,\delta(t - nT_s) \right) * h(t)$$

$$= \sum_{n=-\infty}^{\infty} g[n]\left(\delta(t - nT_s) * h(t)\right) = \sum_{n=-\infty}^{\infty} g[n]\,h(t - nT_s)$$

Here, we use a triangular $h(t)$ as an example.



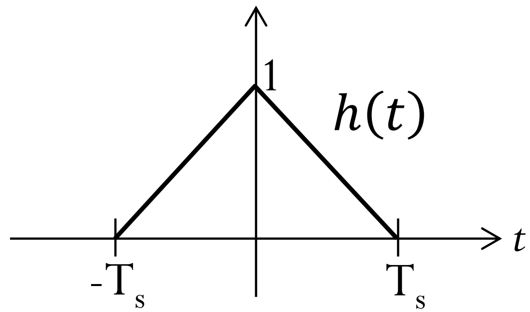
Note that, for this example, the final result is equivalent to "connecting the dots".



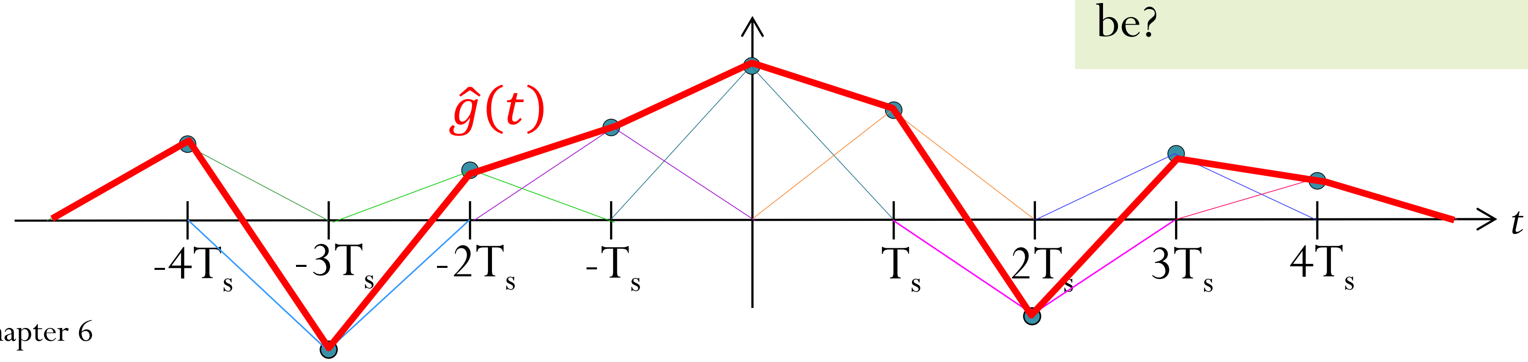
Reconstruction and Convolution

$$\hat{g}(t) = g_\delta(t) * h(t) = \left(\sum_{n=-\infty}^{\infty} g[n]\,\delta(t - nT_s) \right) * h(t)$$

$$= \sum_{n=-\infty}^{\infty} g[n]\left(\delta(t - nT_s) * h(t)\right) = \sum_{n=-\infty}^{\infty} g[n]\,h(t - nT_s)$$

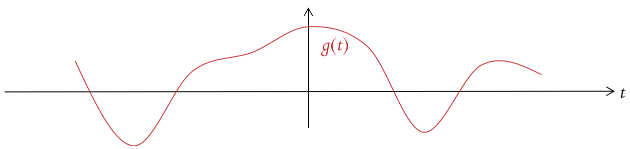


Of course, we know that triangular (linear) interpolation will not give good result. If we want to get $g(t)$ back, what should $h(t)$ be?

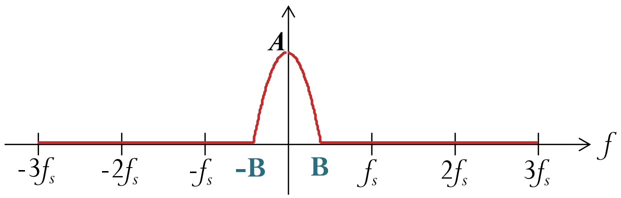


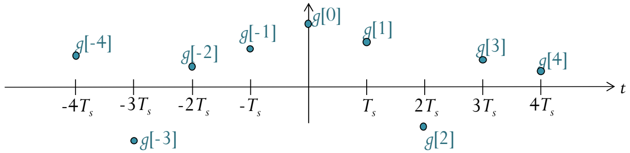
Sampling and Reconstruction

Time Domain | **Frequency Domain**

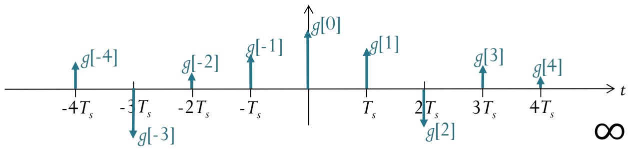


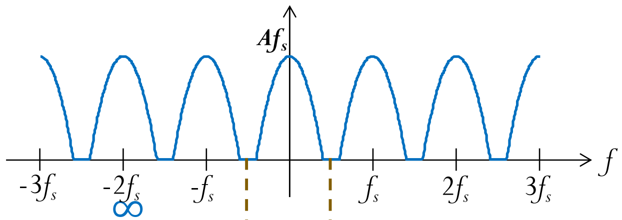
$$g(t) \underset{}{\overset{\mathcal{F}}{\rightleftharpoons}} \mathrm{G}(f)$$



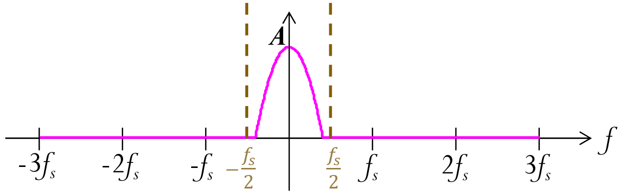


$$g[n] = g(nT_s)$$





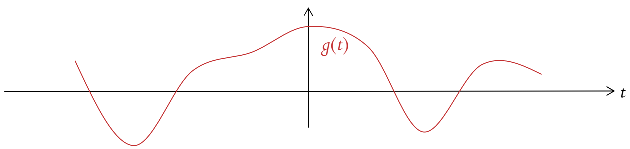
$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g[n]\,\delta(t - nT_s) \overset{\mathcal{F}}{\rightleftharpoons} G_\delta(f) = \sum_{k=-\infty}^{\infty} f_s G(f - kf_s)$$



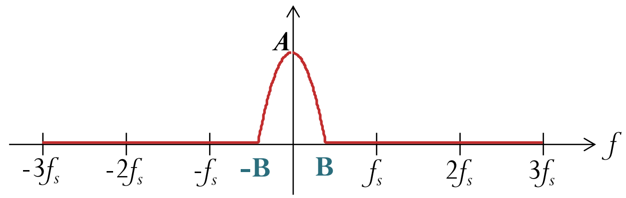
$$\hat{g}(t) \overset{\mathcal{F}}{\rightleftharpoons} \hat{G}(f)$$

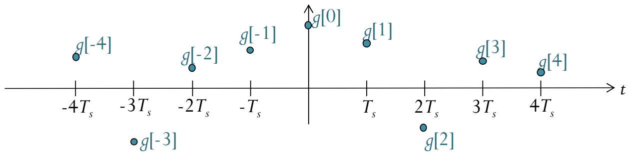
Sampling and Reconstruction

Time Domain | **Frequency Domain**

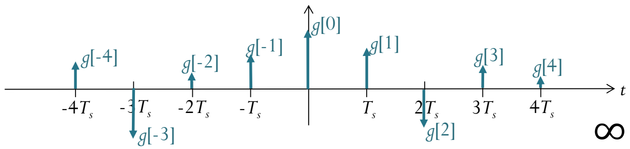


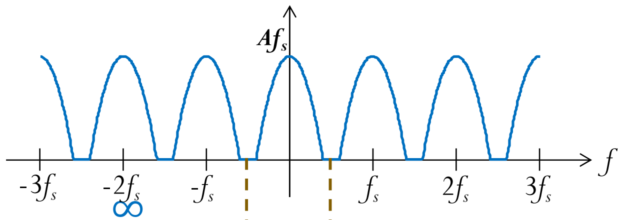
$$g(t) \overset{\mathcal{F}}{\rightleftharpoons} G(f)$$



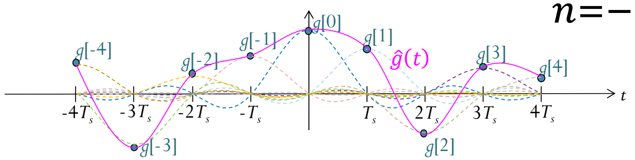


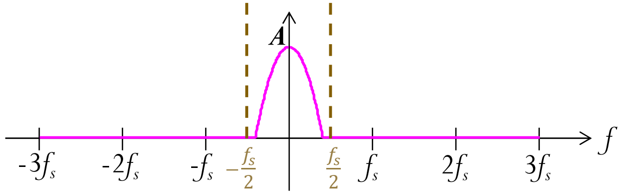
$$g[n] = g(nT_s)$$





$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g[n]\,\delta(t - nT_s) \overset{\mathcal{F}}{\rightleftharpoons} G_\delta(f) = \sum_{k=-\infty}^{\infty} f_s G(f - kf_s)$$

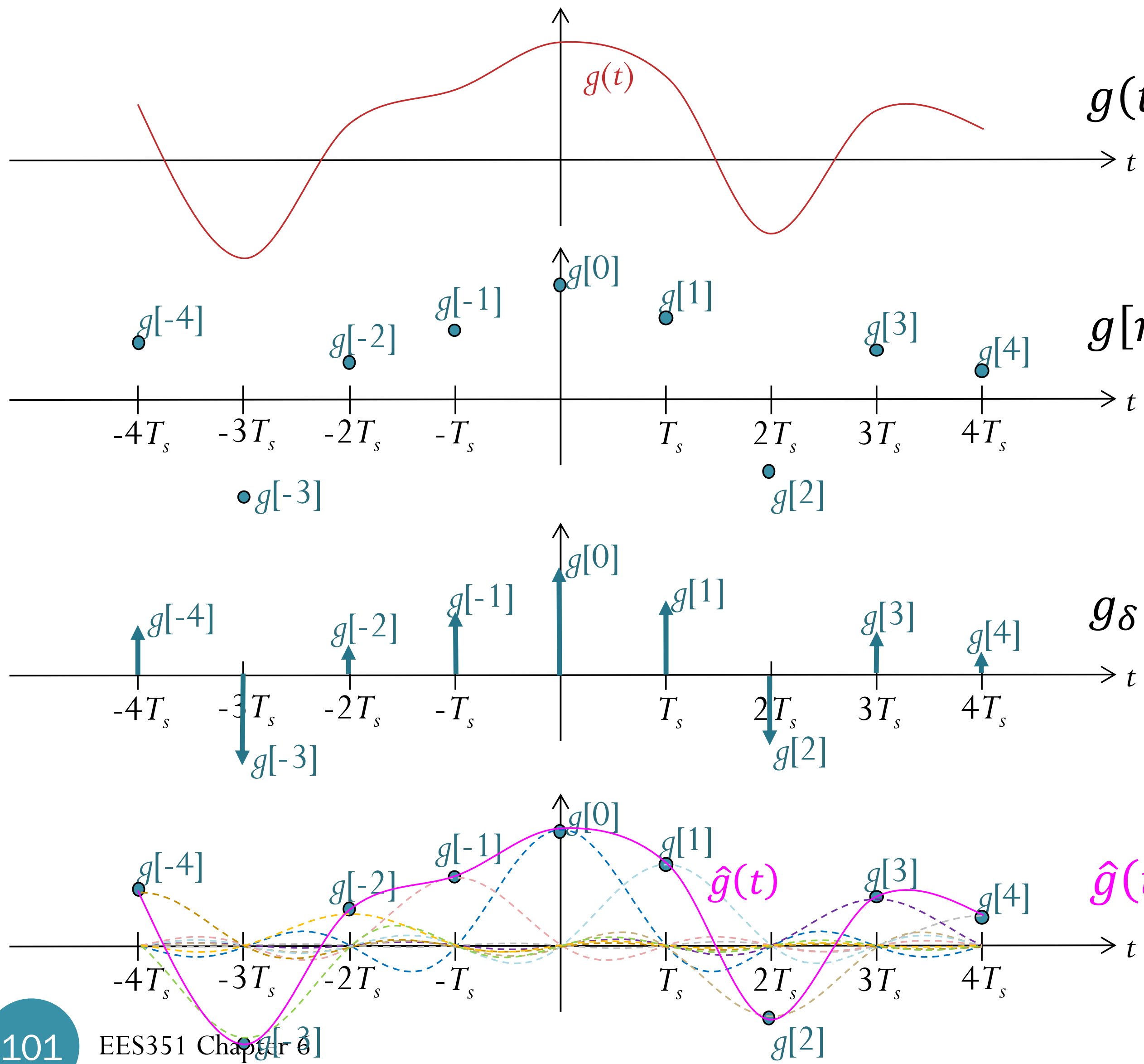




$$\hat{g}(t) = \sum_{n=-\infty}^{\infty} g[n]\,\mathrm{sinc}\big(\pi f_s(t - nT_s)\big) \overset{\mathcal{F}}{\rightleftharpoons} \hat{G}(f)$$

Time Domain

Sampling and Reconstruction



$g(t)$

$g[n] = g(nT_s)$

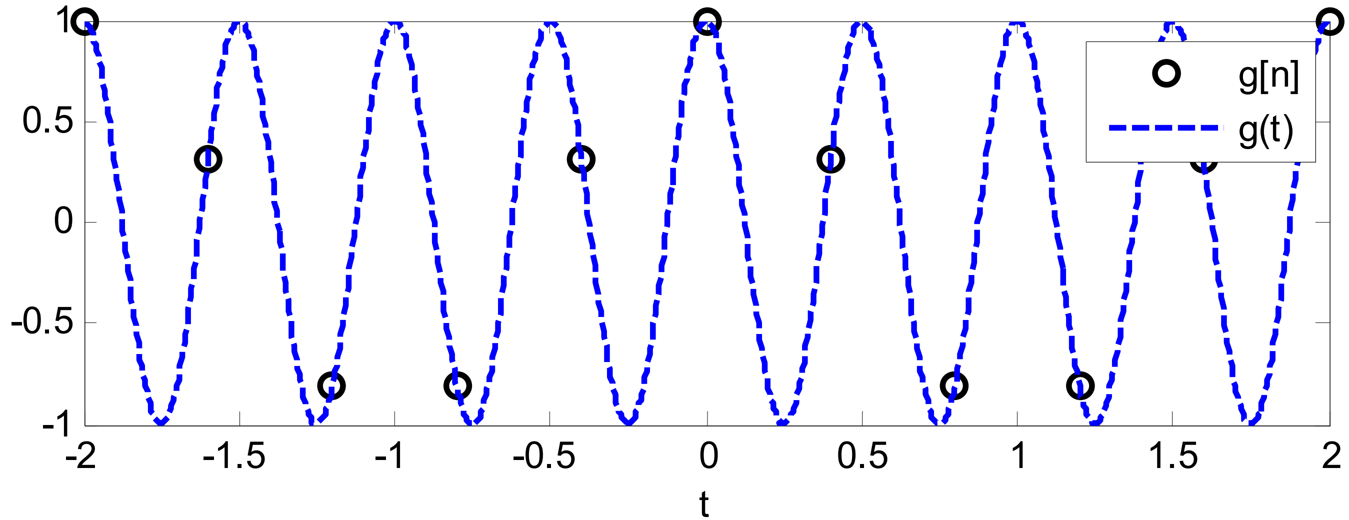
$g_\delta(t) = \sum_{n=-\infty}^{\infty} g[n]\,\delta(t - nT_s)$

$\hat{g}(t) = \sum_{n=-\infty}^{\infty} g[n]\,\mathrm{sinc}\big(\pi f_s (t - nT_s)\big)$

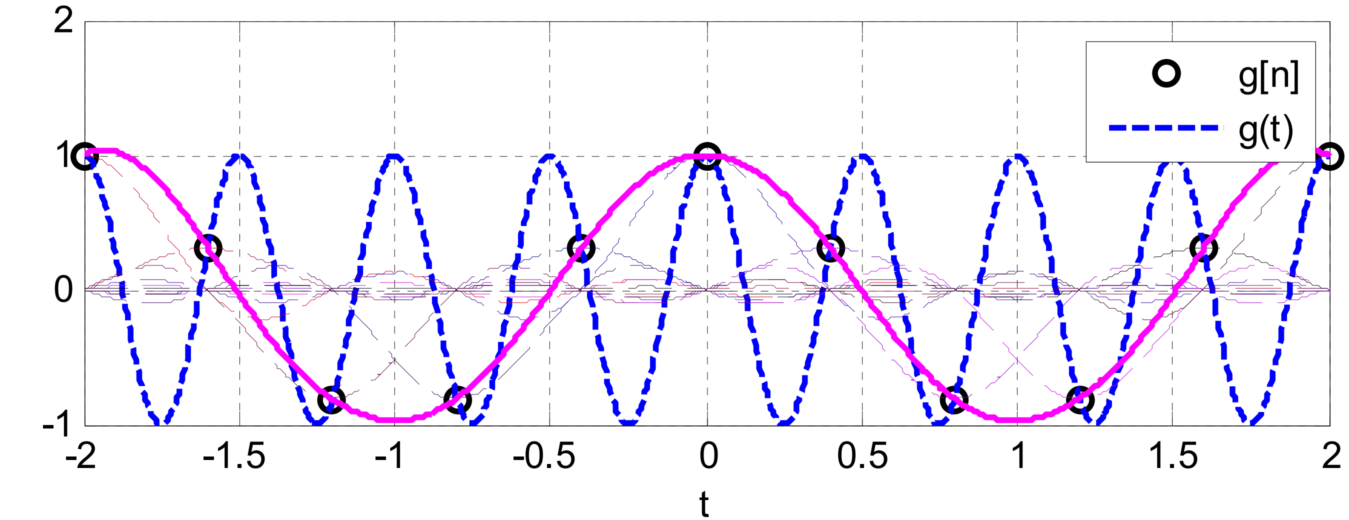
[Example 6.34]

Reconstruction of $\cos(2\pi(2)t)$

$T_s = 0.4$



[Upper plot in Figure 53.]



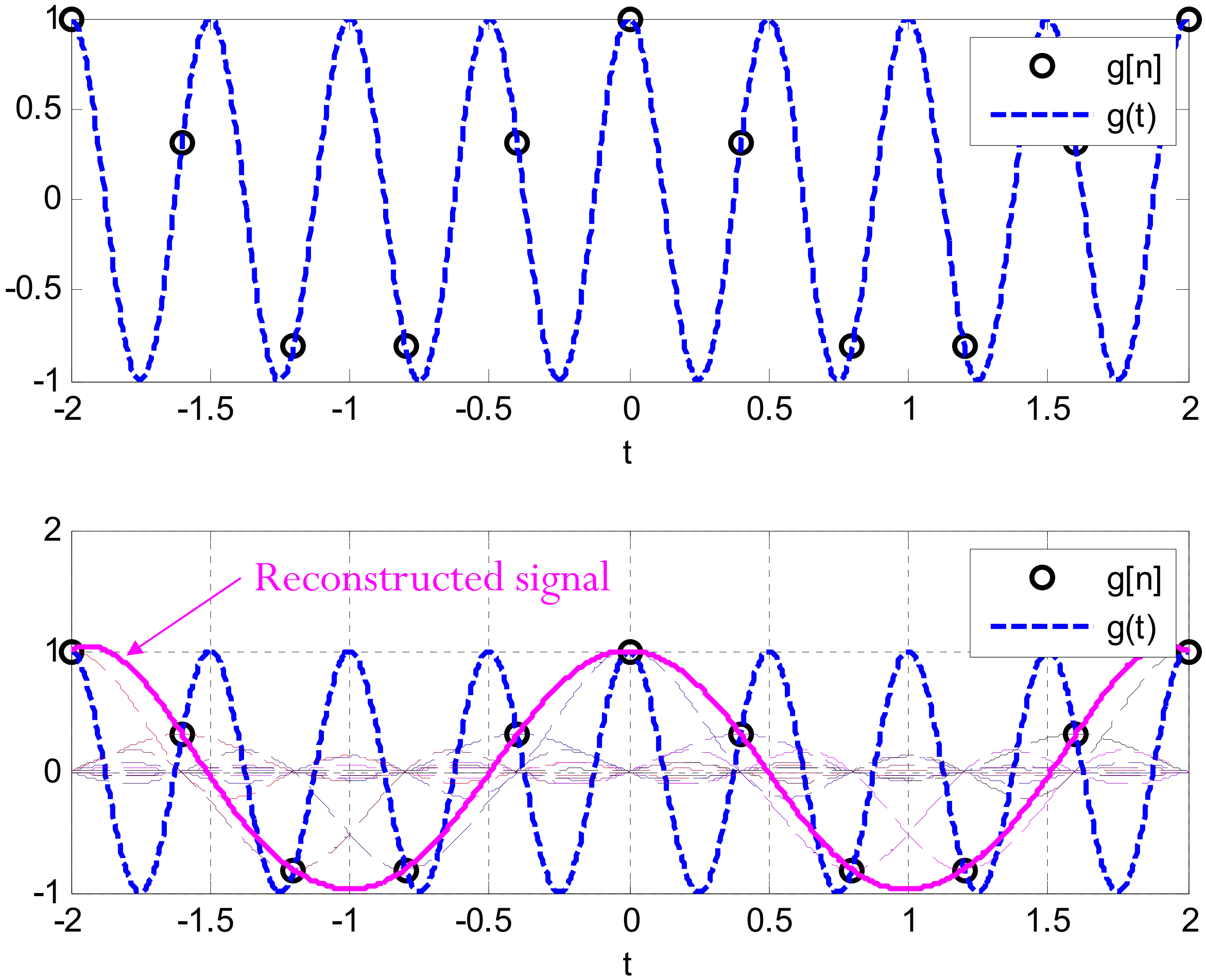
[Example 6.34]

B = 2 Hz.

Reconstruction of $\cos(2\pi(2)t)$

$T_s = 0.4$

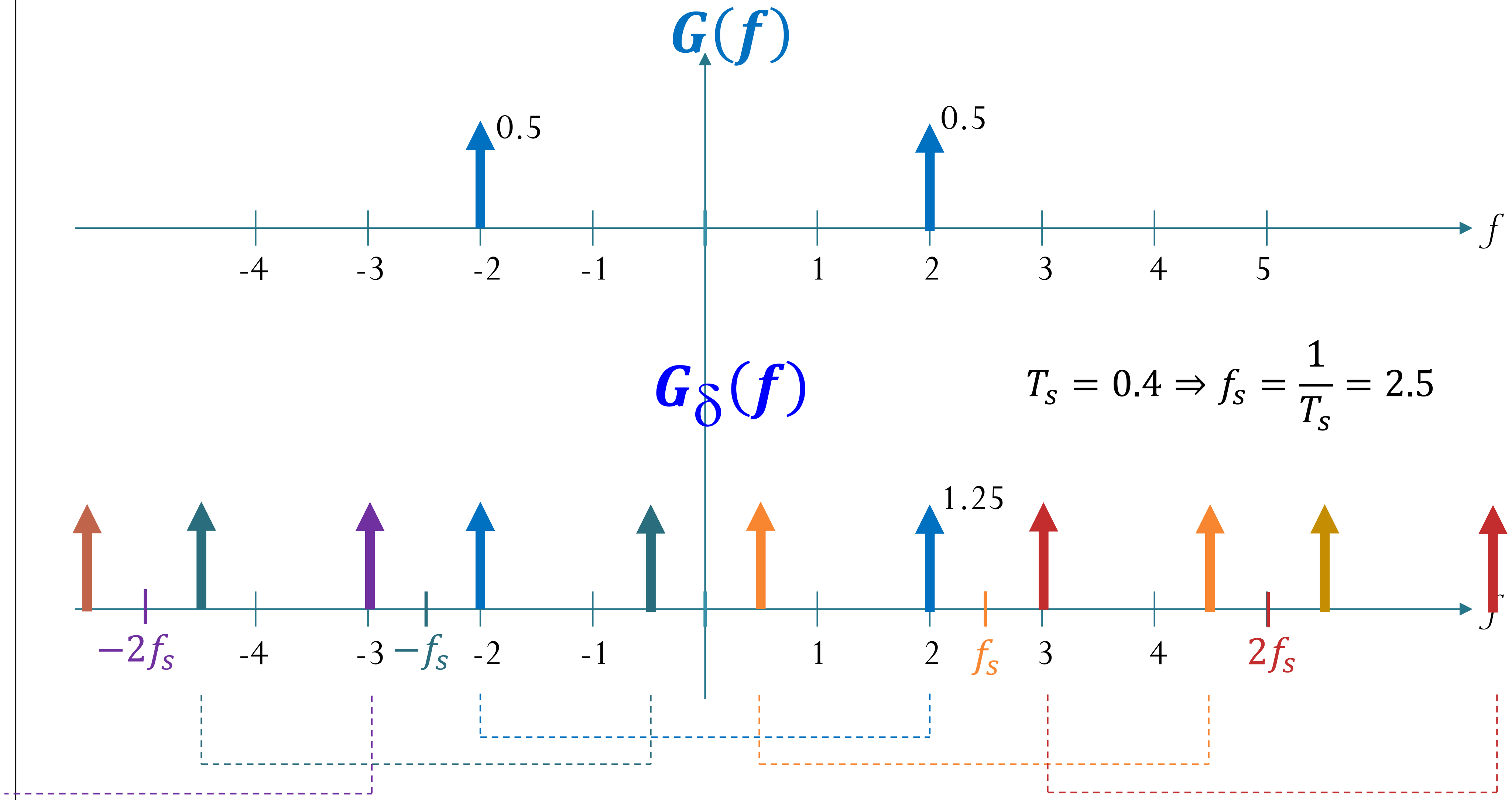
$f_s = 1/0.4$
$= 2.5$ [Sa/s]



[Upper plot in Figure 53.]

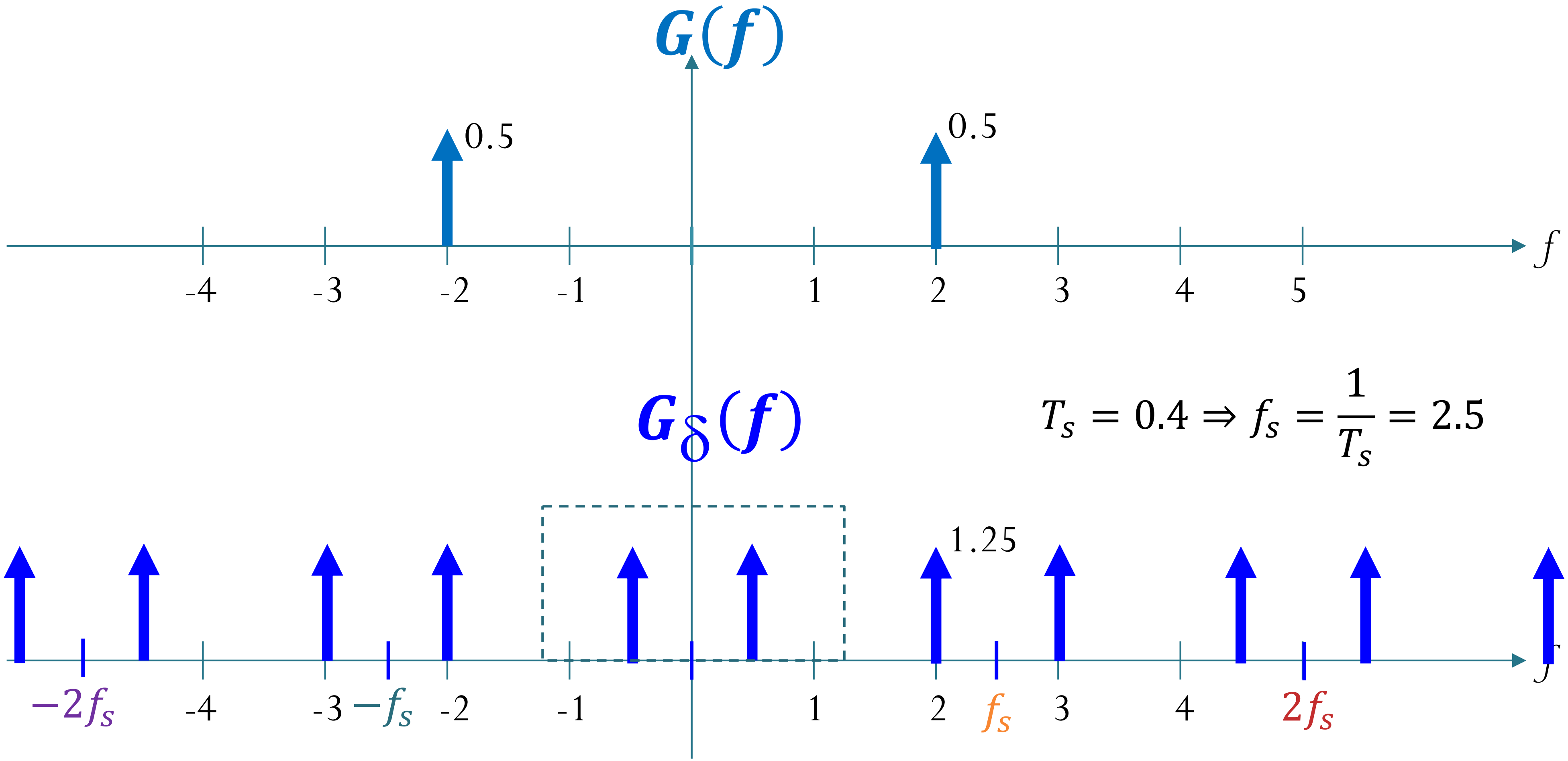
$f_s < 2B \Rightarrow$ the reconstructed signal is different from the original signal.

$G_\delta(f)$ when $g(t) = \cos(2\pi(2)t)$



$T_s = 0.4 \Rightarrow f_s = \frac{1}{T_s} = 2.5$

Reconstruction of $g(t) = \cos(2\pi(2)t)$



$T_s = 0.4 \Rightarrow f_s = \frac{1}{T_s} = 2.5$

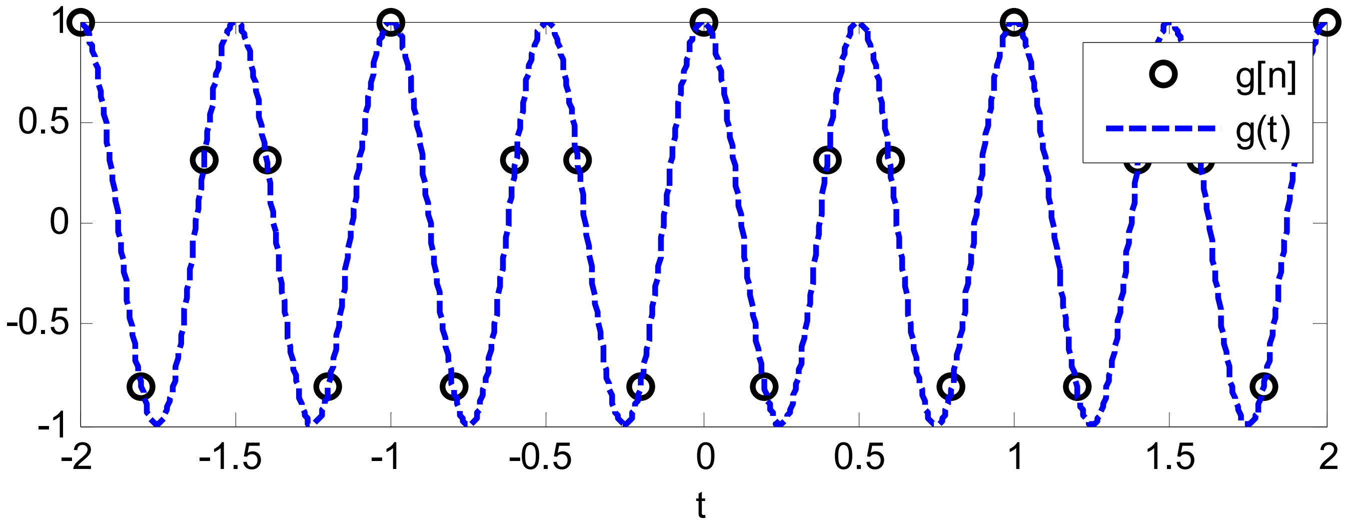
$\hat{g}(t) = \cos(2\pi(0.5)t)$

B = 2 Hz.

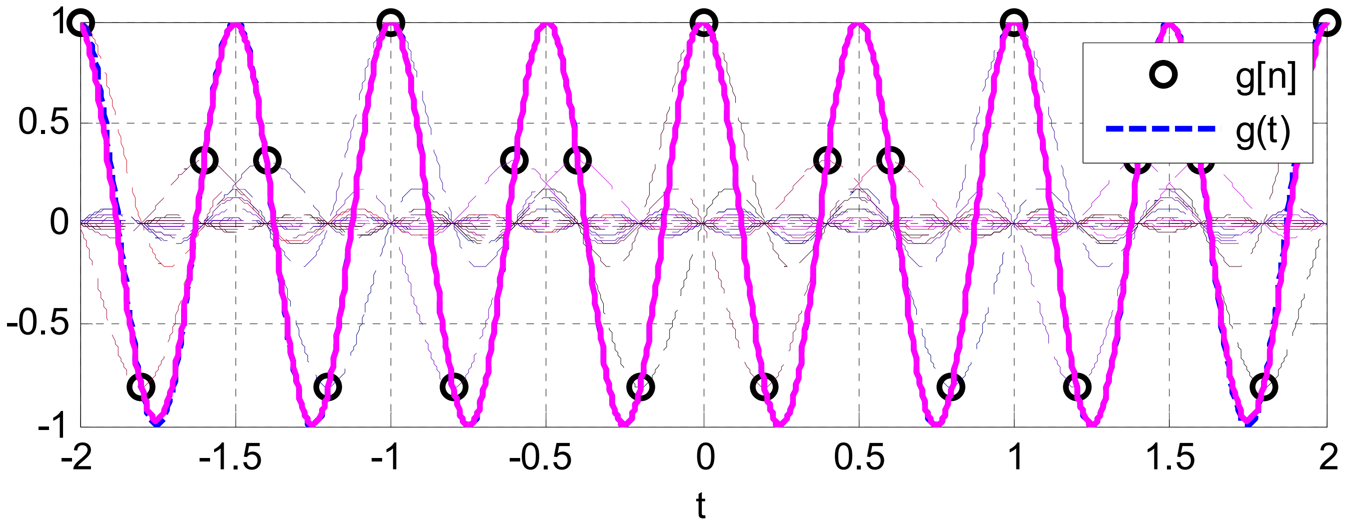
Reconstruction of $\cos(2\pi(2)t)$

$T_s = 0.2$

$f_s = \frac{1}{0.2} = 5$ [Sa/s]



[Lower plot in Figure 53.]

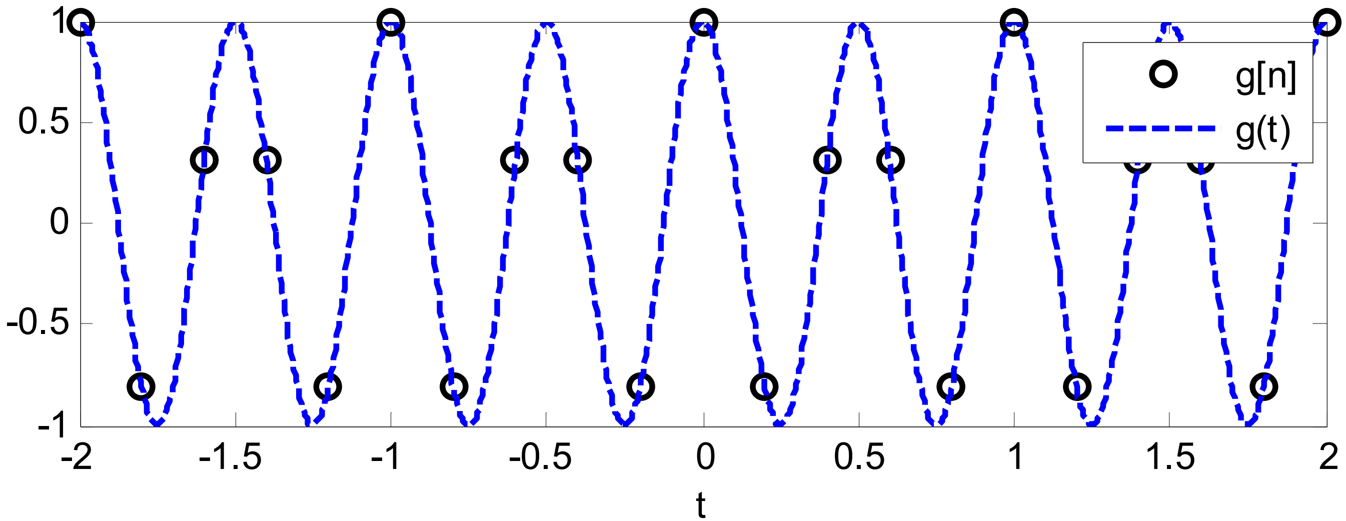


Reconstruction of $\cos(2\pi(2)t)$

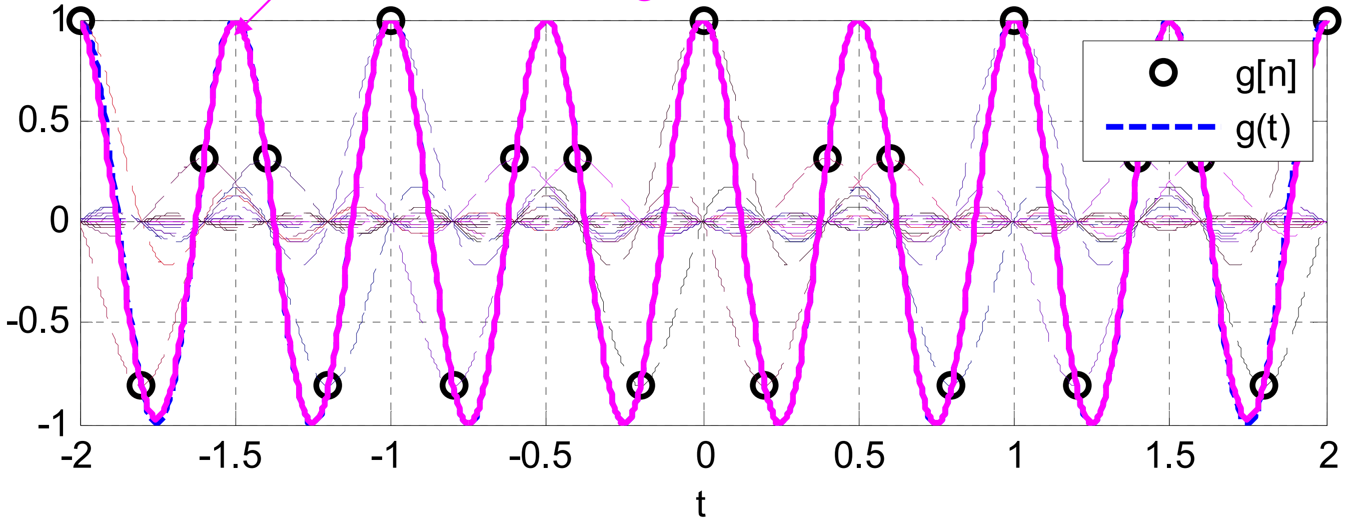
B = 2 Hz.

$T_s = 0.2$

$f_s = \frac{1}{0.2} = 5$ [Sa/s]



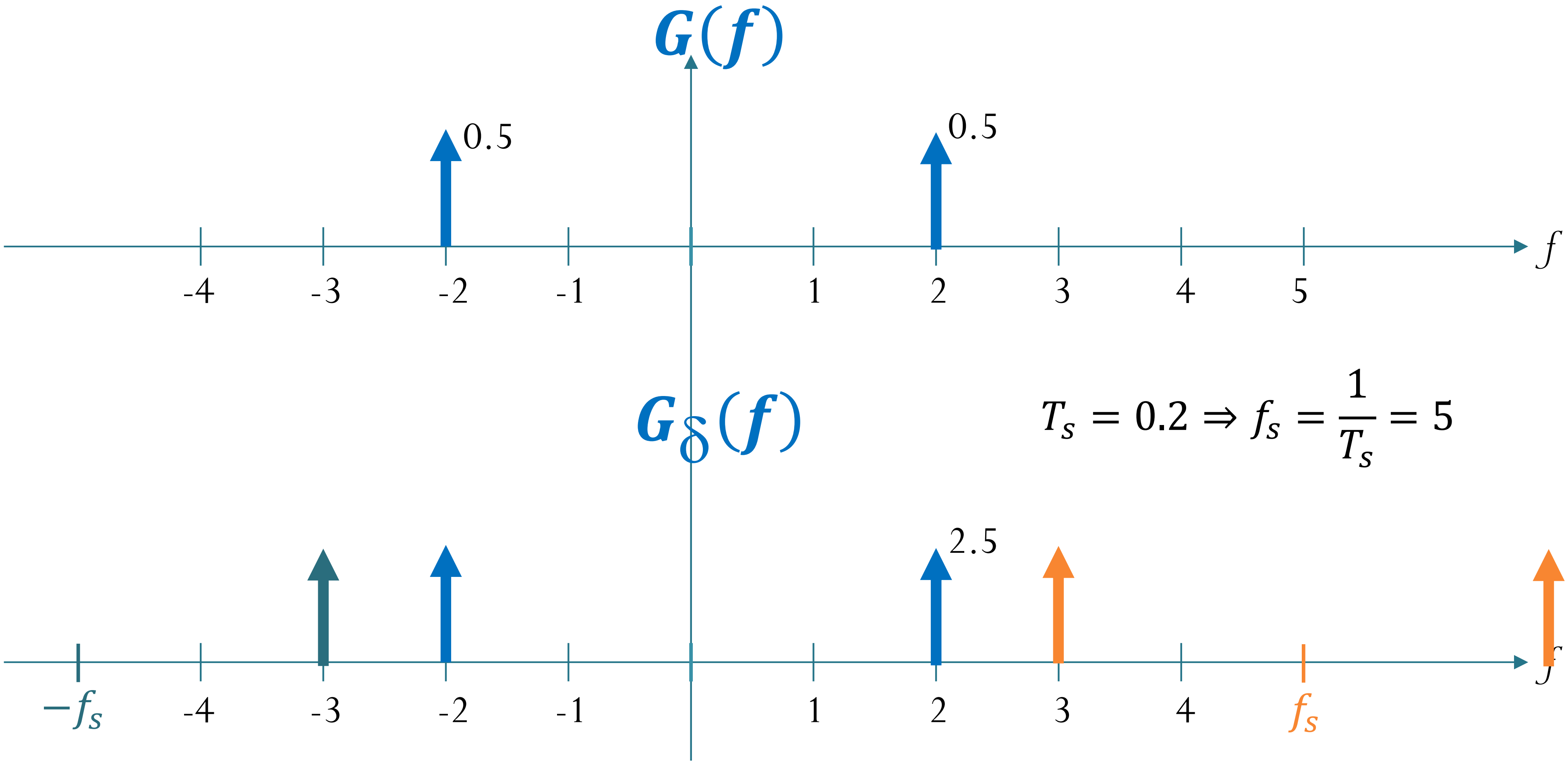
Reconstructed signal



[Lower plot in Figure 53.]

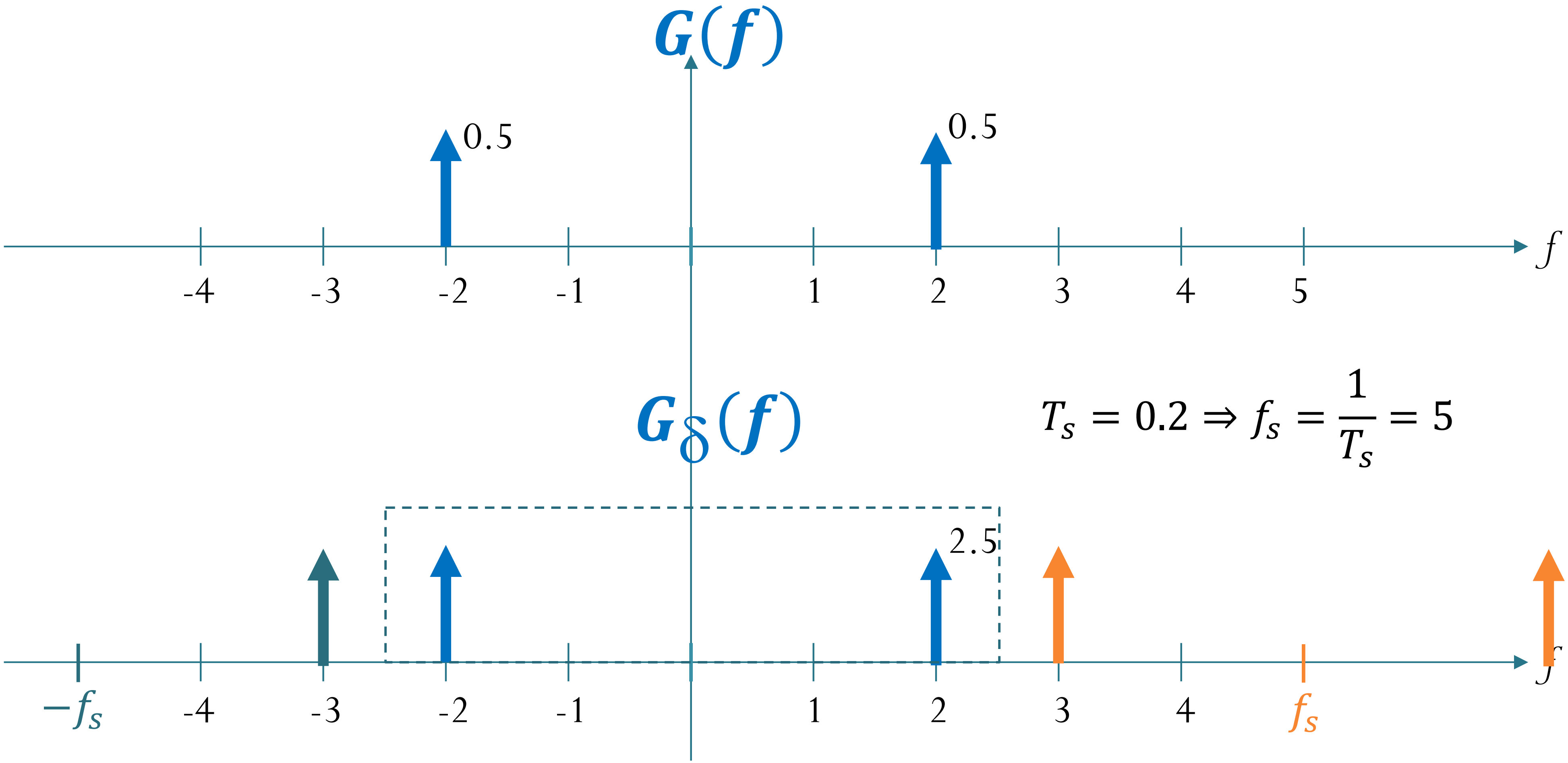
$f_s > 2B \Rightarrow$ the **reconstructed signal** is "the same" as the **original signal.**

$G_\delta(f)$ when $g(t) = \cos(2\pi(2)t)$



$$T_s = 0.2 \Rightarrow f_s = \frac{1}{T_s} = 5$$

Reconstruction of $g(t) = \cos(2\pi(2)t)$



$T_s = 0.2 \Rightarrow f_s = \frac{1}{T_s} = 5$

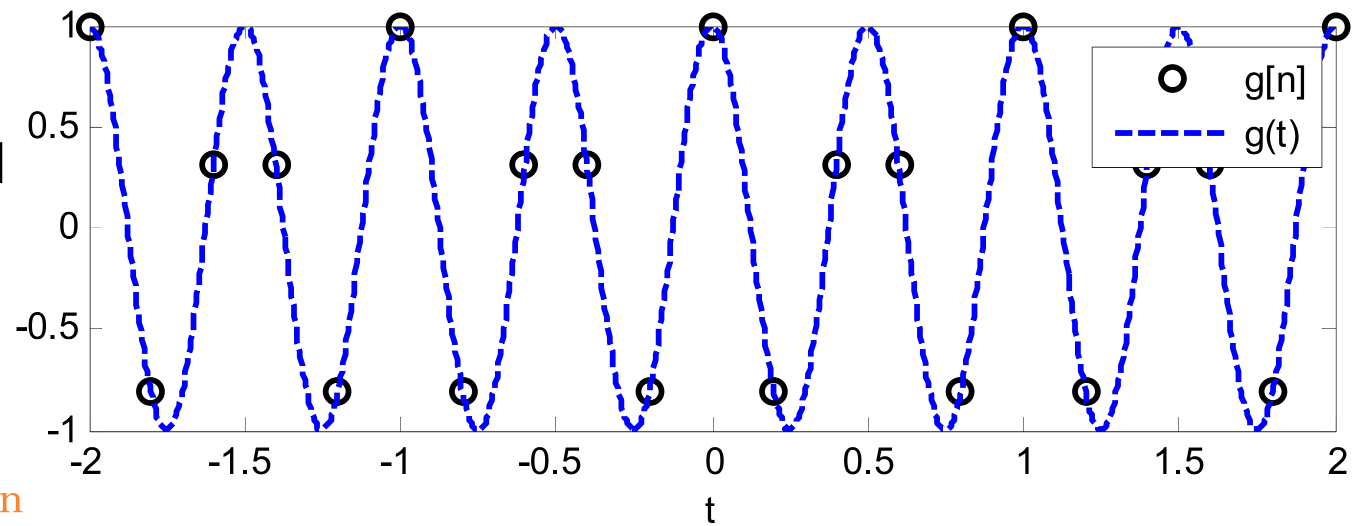
$\hat{g}(t) = \cos(2\pi(2)t)$

B = 2 Hz.

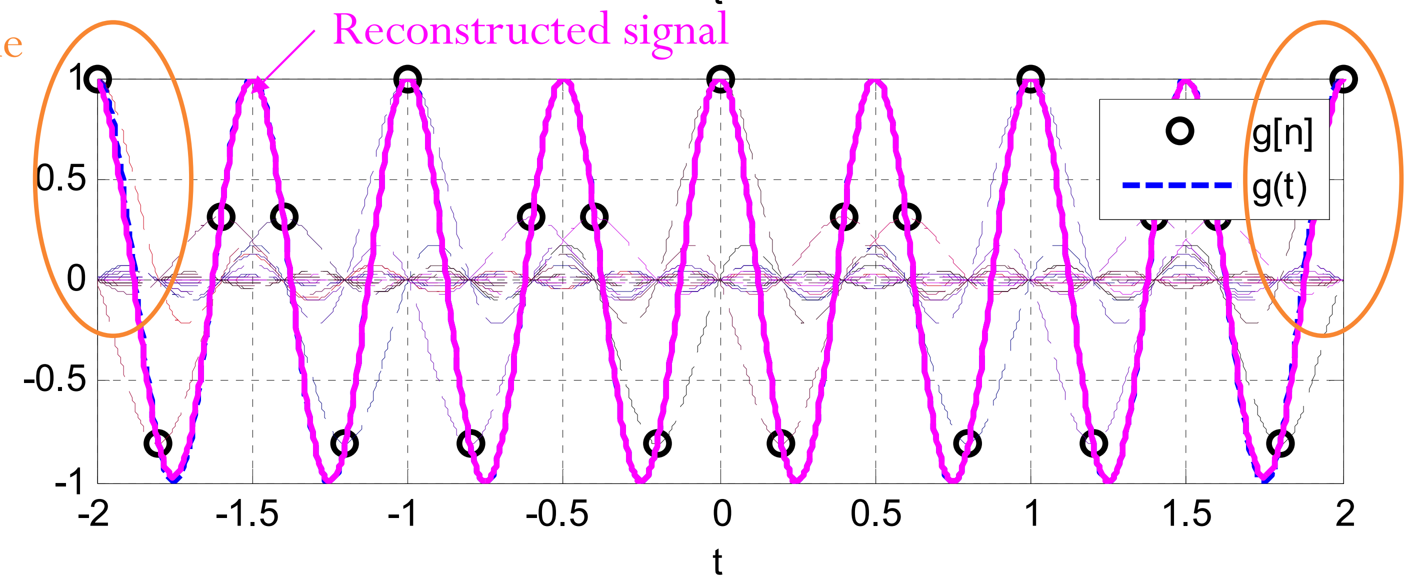
Reconstruction of $\cos(2\pi(2)t)$

$T_s = 0.2$

$f_s = \frac{1}{0.2} = 5$ [Sa/s]



Some reconstruction error is visible at the boundaries because we did not use g[n] for n beyond ±2 in the reconstruction here.



$f_s > 2B \Rightarrow$ the reconstructed signal is "the same" as the original signal.